The Feedback Resistor Is Your Noise Floor

Why a noisy photodiode measurement is usually not the photodiode

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Abstract.
A photodiode receiver that is noisier than expected sends most engineers looking for a detector with lower dark current or a lower noise equivalent power. In a transimpedance front end that is almost never where the noise comes from. Three sources set the input-referred noise: the Johnson noise of the feedback resistor, the shot noise of the photocurrent and dark current, and the amplifier's voltage noise acting against the total capacitance at the summing node. For a representative receiver built around a 100 mm^2 silicon photodiode, the amplifier term supplies 84% of the mean-square noise in a 30 kHz band, the feedback resistor 11%, and the detector's shot noise 4.4%. Removing the dark current entirely would improve the result by 1.1%. The capacitance in the dominant term is the bias-dependent junction capacitance of note 03, so unbiasing the detector raises the total noise by 2.36× and lowers the achievable bandwidth at the same time. Noise, bandwidth and stability are one choice, made by selecting the feedback resistor and the capacitance it works against.

1. The wrong suspect

A photodiode is a current source, and a current source has to be read by something. In nearly every precision measurement that something is a transimpedance amplifier: an operational amplifier with a resistor R_f from its output back to its inverting input, holding the diode at a virtual ground and converting its photocurrent to a voltage $I_{ph}R_f$. When the measured signal is noisier than the budget allowed, the detector is the natural suspect, because it is the only component whose datasheet advertises a noise figure. Dark current and noise equivalent power are printed there; the amplifier's contribution is not.

The suspicion is misplaced. A detector datasheet characterizes the detector alone, into an ideal load. The receiver's noise is set by three sources acting together, and in a transimpedance front end two of them belong to the amplifier and its feedback network. A textbook worked example makes the point without any appeal to a particular part: for a silicon photodiode reading 5 nA of photocurrent into a 1 MΩ sampling resistor, the shot noise contributed by the detector is 0.047 nA while the thermal noise contributed by the resistor is 1.29 nA, a factor of 27 larger [2].

2. Three noise sources, referred to the input

Noise contributions are only comparable once they are expressed in the same units at the same point in the circuit. The natural point is the summing node, where the photocurrent enters, and the natural unit is a current spectral density in amperes per root hertz. Every term below is referred there, so each can be compared directly against the signal photocurrent and the three combine in quadrature, being statistically independent:

$$i_n^2(f) = i_R^2 + i_{\mathrm{sh}}^2 + i_e^2(f) \tag{1}$$

2.1 *Johnson noise of the feedback resistor*

Any resistance at temperature T carries a white noise current whose mean square is $4kT/R$ per unit bandwidth, independent of what the resistor is made of [3, 5]. The feedback resistor sits directly across the summing node, so its noise current adds to the photocurrent with no scaling at all:

$$i_R = \sqrt{\frac{4kT}{R_f}} \tag{2}$$

The square root is the whole reason large feedback resistors are used. Signal grows in proportion to R_{f} while this noise term falls as its square root, so the signal-to-noise ratio of a Johnson-limited receiver improves as $\sqrt{R_{\mathrm{f}}}$. Section 5 puts numbers on how far that improvement runs before the other two terms end it.

2.2 *Shot noise of the photocurrent and the dark current*

Current in a junction is carried by discrete charges arriving at random times, so any current through the diode fluctuates about its mean. The photocurrent and the dark current are statistically independent processes, so their mean squares add [1]:

$$i_{\mathrm{sh}} = \sqrt{2q\left(I_{\mathrm{ph}} + I_d\right)} \tag{3}$$

This is the only term that belongs to the detector, and it is the only one that a lower dark current improves. Note also that it is white and that the photocurrent appears in it: a receiver at high signal level is shot-noise limited whatever its amplifier does, which is the regime detector datasheets are written for and is not the regime a low-light measurement operates in.

2.3 *Amplifier voltage noise against the input capacitance*

The third term is the one that surprises people. An operational amplifier presents an input voltage noise e_{n} in series with its input, and feedback forces that voltage to appear across whatever impedance connects the summing node to ground. At the summing node that impedance is almost entirely capacitive, and a capacitor's admittance rises with frequency. The resulting input-referred current noise is

$$i_e(f) = e_n\sqrt{\left(\frac{1}{R_f}\right)^2 + \left(2\pi f C_T\right)^2} \tag{4}$$

where C_T is the total capacitance at the summing node: the diode's junction capacitance, the amplifier's own input capacitance, and board stray, all in parallel [1]. Below a corner frequency $1/(2\pi R_f C_T)$ the first term governs and Eq. (4) is flat at e_n/R_f, usually negligible. Above it the second term governs and the noise rises in direct proportion to frequency. Referred to the output the same behaviour is called noise gain peaking: the amplifier's own voltage noise reaches the output multiplied by a gain that starts at unity, begins climbing at the zero $1/(2\pi R_f C_T)$, and levels off only when the feedback capacitance introduced in Section 7 takes over [4].

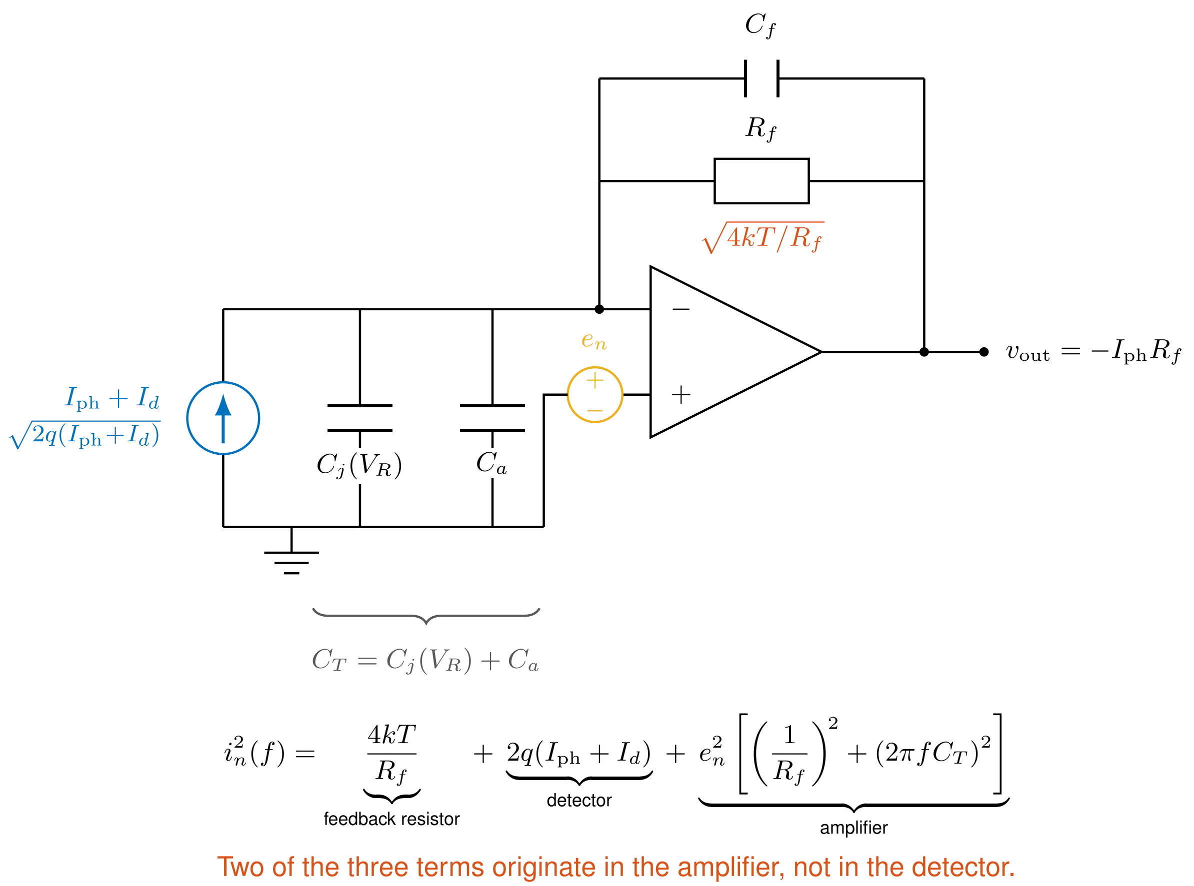

Figure 1. The transimpedance front end with each noise source drawn where it originates rather than where it is measured. The detector contributes one generator, the shot noise of the total diode current. The other two belong to the feedback resistor and to the amplifier, and the amplifier's contribution is the only one that depends on frequency, because it works against the capacitance C_T at the summing node rather than against a resistance.

What is left out. The amplifier's input current noise is omitted from Eq. (1). For the JFET-input devices used with photodiodes it is a few femtoamps per root hertz, and at 10 fA/√Hz it raises the 1 kHz total of the worked example below by 0.21% in quadrature. The diode's shunt resistance is also omitted: it appears in parallel with R_f in Eq. (2), and for a reverse-biased silicon photodiode it is orders of magnitude the larger of the two, so it moves nothing. Flicker noise in the amplifier's input stage matters below roughly a hundred hertz and is a separate design problem from the one treated here.

3. A worked receiver

Take a detector with the characteristics developed in note 03 of this series: a 100 mm² silicon photodiode whose junction capacitance is a published 1.0 nF at zero bias and follows $C_j(V_R) = C_j(0)\sqrt{(V_{bi}/(V_{bi} + V_R))}$ with an effective V_{bi} = 0.87 V, giving 385 pF at 5 V reverse bias. Read it with a JFET-input amplifier of e_n = 8 nV/√Hz and 10 MHz gain-bandwidth product, and allow 15 pF for the amplifier's input capacitance together with board stray. The total at the summing node is then 400 pF. Set R_f = 1 MΩ, take a photocurrent and a dark current of 10 nA each, and work at 298 K.

Equations (2) and (3) give 0.128 pA/√Hz for the feedback resistor and 0.0801 pA/√Hz for the shot noise, a ratio of 1.60 before the amplifier is considered at all. Equation (4) is flat at 0.0080 pA/√Hz until its zero at 398 Hz, then climbs, and it overtakes the feedback resistor at

$$f_x = \frac{\sqrt{4kT\ /\ R_f}}{2\pi e_n C_T} \tag{5}$$

which for these values is 6.4 kHz. Figure 2 and Table 1 carry the three terms across the band.

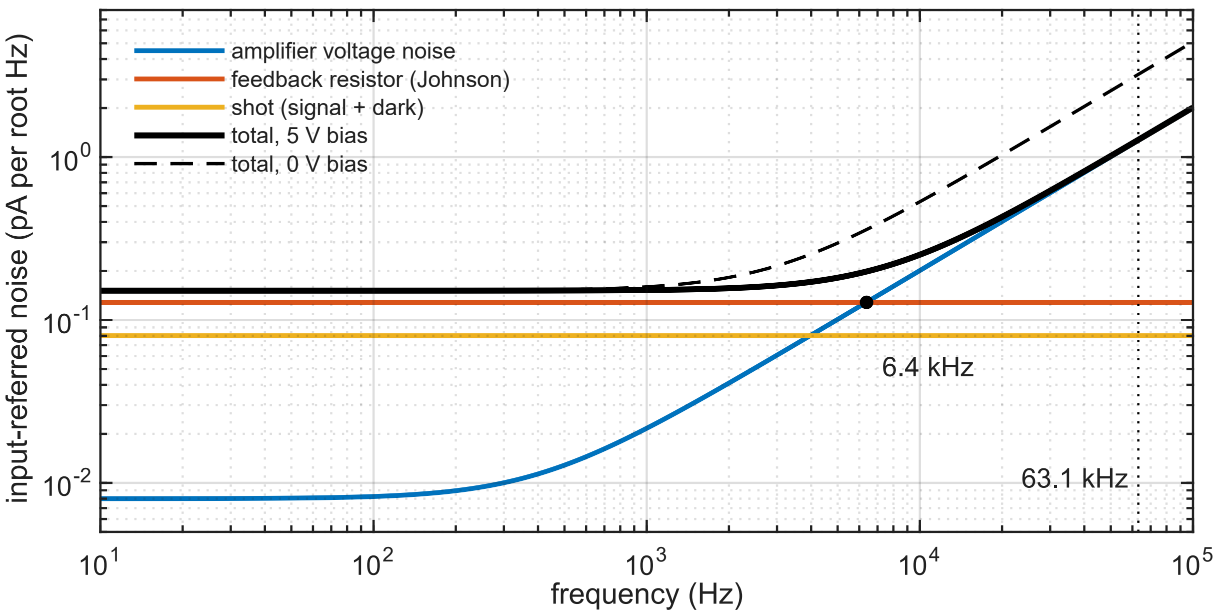

Figure 2. The three contributions of Eqs. (2) to (4) and their quadrature sum, for the receiver of Section 3. Below the crossover at 6.4 kHz the total is set by the feedback resistor, with the detector's shot noise a factor of 1.60 below it; above the crossover it is set by the amplifier working against C_T, and the detector has stopped mattering entirely. The dashed curve is the same receiver with the diode unbiased: nothing changed but C_j, and the rising region moved down in frequency by the ratio of the two capacitances.

Table 1. Input-referred noise density of the worked receiver at 5 V bias, in pA/√Hz. The first two columns are white; only the amplifier term moves with frequency, and it moves fast enough to dominate everything above 6.4 kHz. The last row is the closed-loop bandwidth derived in Section 7.

Frequency	i_R, Eq. (2)	i_{sh}, Eq. (3)	i_e, Eq. (4)	total, Eq. (1)
100 Hz	0.128	0.0801	0.00825	0.151
1 kHz	0.128	0.0801	0.0216	0.153
10 kHz	0.128	0.0801	0.201	0.252
63.1 kHz	0.128	0.0801	1.268	1.277

4. From noise density to a measurement

A density is not a measurement. Integrating Eq. (1) over a brick-wall band from zero to B gives the rms noise current the measurement will show. The two white terms integrate to B, and the frequency-proportional term integrates to $B^3/3$:

$$I_n^2 = \left(\frac{4kT}{R_f} + 2q\left(I_{\mathrm{ph}} + I_d\right) + \frac{e_n^2}{R_f^2}\right)B + \frac{e_n^2\left(2\pi C_T\right)^2 B^3}{3} \tag{6}$$

The cube is what makes the amplifier term decisive. Over a 30 kHz band the worked receiver delivers 65.8 pA rms, of which the amplifier term supplies 84% of the mean square, the feedback resistor 11%, and the detector's shot noise 4.4%. The residue is the flat $e_\mathrm{n}/R_\mathrm{f}$ term.

The test that settles it. Set the dark current of the worked receiver to zero, which is the best any detector selection can possibly achieve, and the total falls from 65.8 to 65.0 pA rms, an improvement of 1.1%. Halving C_T instead improves it by a factor of 1.65. Capacitance, not dark current, is the detector specification that governs a transimpedance receiver.

5. Signal to noise against feedback resistance

Because every term of Eq. (6) is referred to the input, the signal-to-noise ratio is simply $I_\mathrm{ph}/I_\mathrm{n}$, and R_f cancels out of the signal. Only two of the four terms depend on R_f at all, and both fall as it rises, so signal to noise climbs with R_f and then stops. While the Johnson term dominates, the climb is the $\sqrt{R_\mathrm{f}}$ law of Section 2.1. It ends where the Johnson term drops to the level of the amplifier term, at

$$R_f^{\star} = \frac{12kT}{e_n^2\left(2\pi C_T\right)^2 B^2} \tag{7}$$

which for the worked receiver is 136 kΩ. Figure 3 shows the consequence. Raising R_f from 10 kΩ to 100 kΩ improves signal to noise from 37.2 to 106, a factor of 2.84 for a decade of resistance, short of the √10 a purely Johnson-limited receiver would return because the other two terms are already contributing at 100 kΩ. Raising it a further decade, from 1 MΩ to 10 MΩ, moves it from 152 to 161, a gain of 5.6%, against a ceiling of 162 that no feedback resistor can pass. The last decade of resistance costs a decade of bandwidth (Section 7) and buys almost nothing.

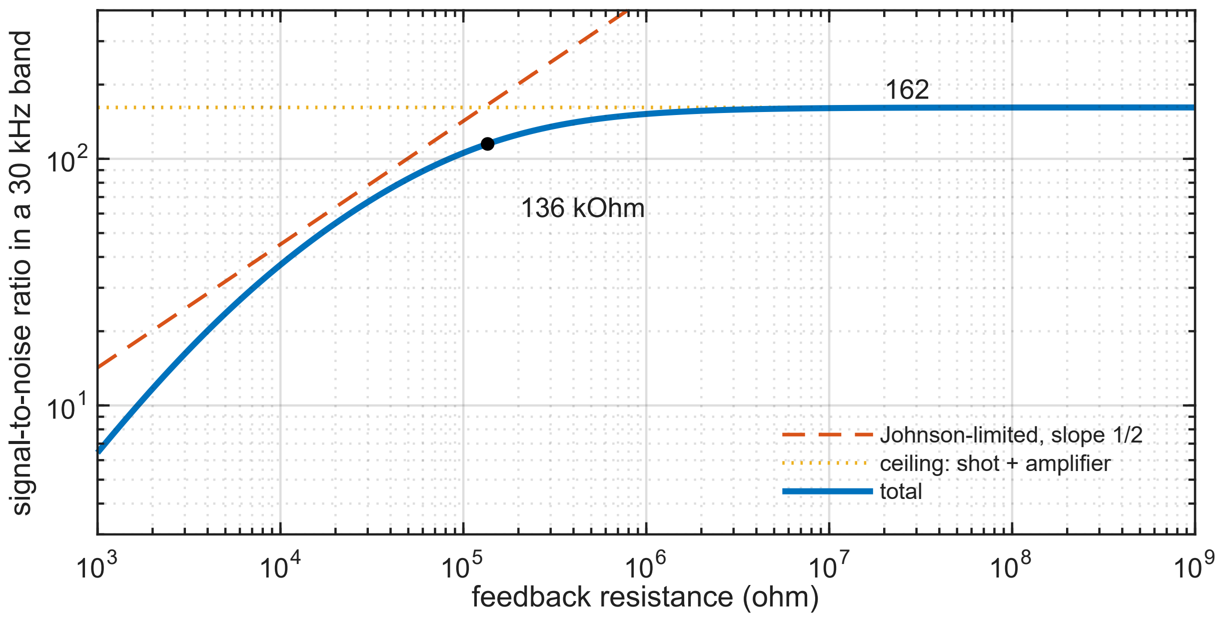

Figure 3. Signal to noise against feedback resistance for the receiver of Section 3, integrated over a 30 kHz band. The dashed line is the Johnson-limited asymptote, of slope one half on these axes; the dotted line is the ceiling set by the two terms that do not contain R_f. The marker at $R_f^{\star}$ = 136 kΩ is where the two curves have already parted: beyond it the square-root return is spent, and the remaining distance to the ceiling is 1.4×.

A second crossover is worth knowing even though it rarely binds. The Johnson term of Eq. (2) equals the shot term of Eq. (3) at $R_f = 2kT/q(I_{ph} + I_d)$, which for a total diode current of 20 nA is 2.6 MΩ. A receiver whose feedback resistance exceeds that value is shot-noise limited at low frequency and genuinely does benefit from a lower dark current. In a wideband receiver that condition is almost never met, because the capacitance term has already taken over long before the resistance gets that large.

6. The capacitance is the one from note 03

C_T in Eq. (4) is not a constant of the detector. Its dominant part is the junction capacitance, which tracks the depletion width and therefore falls as the square root of applied reverse bias. Note 03 of this series develops that dependence and tabulates it for the same 100 mm^2 device used here: 1.0 nF at zero bias against 385 pF at 5 V. Adding the same 15 pF of amplifier and stray capacitance gives 1015 pF at zero bias against 400 pF at 5 V, a ratio of 2.54.

Nothing else in the receiver changes, and yet every quantity in this note moves. The crossover of Eq. (5) falls from 6.4 kHz to 2.5 kHz, so the amplifier term takes over 2.54× earlier in frequency. The integrated noise of Eq. (6) over the same 30 kHz band rises from 65.8 to 155 pA rms, a factor of 2.36. The dashed curve of Figure 2 is that shift drawn out.

The direction matters as much as the size. Photovoltaic operation at zero bias is chosen precisely to lower dark current and flicker noise, and against the detector's own shot noise it does exactly that. Against the receiver's noise it does the opposite, because it triples the capacitance the amplifier's voltage noise works against. Note 03 showed that reducing bias costs bandwidth; the same capacitance, in the same direction, costs noise as well. An engineer who unbiases a large-area detector to quiet it down has made the measurement noisier and slower at once.

7. Noise, bandwidth and stability are one choice

The rise in Eq. (4) cannot continue without limit, and the element that arrests it is the same one that sets the bandwidth. A feedback capacitance C_f across R_f puts a pole in the transimpedance at $1/(2\pi R_\mathrm{f} C_\mathrm{f})$ and flattens the noise gain above it at $1 + C_\mathrm{T}/C_\mathrm{f}$ [4]. It is not optional: without it the rising noise gain meets the amplifier's falling open-loop gain at a closing rate that makes the loop unstable, and the circuit peaks or oscillates. Setting the pole where the noise gain meets the open-loop gain gives the largest bandwidth that remains stable,

$$C_f \approx \sqrt{\frac{C_T}{2\pi R_f \mathrm{GBW}}}, \quad f_{3\mathrm{dB}} \approx \sqrt{\frac{\mathrm{GBW}}{2\pi R_f C_T}} \tag{8}$$

For the worked receiver this is 2.52 pF and 63.1 kHz. Three consequences follow immediately, and they are the reason the three quantities cannot be chosen separately.

- The peak noise gain is $1 + C_\mathrm{T}/C_\mathrm{f}$ = 160 here. An amplifier chosen for its 8 nV/√Hz arrives at the band edge having been multiplied by more than two orders of magnitude, which is why Eq. (4) beats a detector specification that never moves.
- Bandwidth falls as $1/\sqrt{R_\mathrm{f}}$, so the decade of resistance that bought 5.6% of signal to noise in Section 5 costs a factor of √10 in bandwidth.
- Bandwidth also falls as $1/\sqrt{C_\mathrm{T}}$, not as $1/C_\mathrm{T}$. Unbiasing the detector takes the worked receiver from 63.1 to 39.6 kHz, a factor of 1.59, which is the square root of the 2.54 capacitance ratio. A resistively loaded detector loses the full factor instead, as note 03 shows.

A practical caution on C_f. A computed 2.52 pF is close to the stray capacitance of the feedback resistor and its board traces, which is commonly a few tenths of a picofarad and is not controlled. At high feedback resistance the compensation capacitor becomes small enough that the layout, not the component, sets the pole. This is a second reason the largest available R_f is the wrong default. It is also why C_f should be added to C_T in Eq. (4) when it is not small; here it is 0.63% of C_T and has been omitted.

8. What this does to a detector specification

Noise equivalent power is the optical power that produces a signal equal to the noise in a 1 Hz band [1]. A detector datasheet computes it from the detector's own noise, which under reverse bias is the shot noise of the dark current. For the device here, that is $\sqrt{(2qI_\mathrm{d})}$ = 0.0566 pA/√Hz, and at a responsivity of 0.430 A/W (the measured value at 800 nm reported in note 02 of this series) an NEP of 0.132 pW/√Hz.

The receiver's NEP is the total of Eq. (1) divided by the same responsivity. At 1 kHz that is 0.355 pW/√Hz, a factor of 2.70 worse than the detector's own figure, and the gap widens across the rest of the band. Two detectors quoted at different NEP will not produce measurements in that ratio unless the receiver has been designed so that the detector still dominates, which for a large-area device means a small feedback resistance, a narrow band, or both. Comparing detectors on NEP alone answers a question about the detectors and not about the instrument.

Verdict. In a photodiode transimpedance receiver the noise floor is the feedback resistor's Johnson noise at low frequency and the amplifier's voltage noise against C_{T} above the crossover of Eq. (5). The detector's shot noise is usually the smallest of the three, and the dark current is the smallest part of that. Budget the receiver, not the detector: compute Eqs. (2) to (4) at the frequencies that matter, raise R_{f} only until Eq. (7) says the square-root return has gone, and then reduce C_{T}, which means a smaller active area first and more reverse bias second. The capacitance that sets the noise is the same bias-dependent capacitance that sets the bandwidth, so the two are bought and lost together.

Notes

Symbols. i_{n} total input-referred noise current density; i_{R}, i_{sh}, i_{e} its feedback-resistor, shot and amplifier terms; I_{n} the same quantity integrated over a band; R_{f}, C_{f} feedback resistance and capacitance; $R_{\mathrm{f}}^{\star}$ the feedback resistance at which the Johnson term falls to the amplifier term; e_{n} amplifier input voltage noise density; C_{j} junction capacitance; C_{a} amplifier input capacitance plus board stray; $C_{\mathrm{T}} = C_{\mathrm{j}} + C_{\mathrm{a}}$ total capacitance at the summing node; I_{ph}, I_{d} photocurrent and dark current; V_{R} reverse bias; V_{bi} built-in potential; B measurement bandwidth; GBW amplifier gain-bandwidth product; f_{x} the Johnson-to-amplifier crossover; f_{3dB} closed-loop bandwidth; k Boltzmann's constant; q elementary charge; T absolute temperature.

Basis. The detector is the 100 mm^2 silicon photodiode of note 03: 1.0 nF published typical junction capacitance at zero bias, following the one-sided abrupt-junction law with an effective V_{bi} = 0.87 V fitted there to two published endpoints, which gives 385 pF at 5 V. The responsivity used for NEP is the 0.430 A/W measured at 800 nm reported in note 02. Everything else is a stated input of the worked example rather than a specification of any product: e_{n} = 8 nV/√Hz, GBW = 10 MHz and 10 fA/√Hz of input current noise, all representative of a low-noise JFET-input operational amplifier; 15 pF of amplifier input plus board stray capacitance; R_{f} = 1 MΩ; $I_{\mathrm{ph}} = I_{\mathrm{d}}$ = 10 nA; B = 30 kHz; T = 298 K. Constants are the SI values $k = 1.380649 \times 10^{-23}$ J/K and $q = 1.602176634 \times 10^{-19}$ C. Equation (6) assumes a brick-wall band, which overstates the amplifier term slightly against a real single-pole roll-off, and assumes B lies inside the closed-loop bandwidth of Eq. (8); at 30 kHz against 63.1 kHz it does. Flicker noise, the diode shunt resistance and the amplifier's input current noise are excluded, with the reason given in Section 2.3.

References

1. S. O. Kasap, *Optoelectronics and Photonics: Principles and Practices*, 2nd ed., Pearson, 2013, ISBN 978-0-273-77417-4, pp. 423–424 (the terminal capacitance, which "should include the capacitance of the buffer amplifier as well", and $R_{\mathrm{F}}C_{\mathrm{t}}$ as the critical time constant of a current-to-voltage converter) and pp. 425–426 (shot noise of the dark current and of the photocurrent, Eqs. 5.12.1–5.12.3; the NEP definition, Eq. 5.12.5).
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